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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

LIE ALGEBROIDS AND GROUPOIDS AND PROPERTIES OF FUNCTORS IN THE FUZZY MODULUS CATEGORY

Specialty: 1210.01 – Topology

Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THE WORK

Relevance and degree of study of the topic. The dissertation work was devoted to the Lie algebroid and groupoids and properties of functors in the fuzzy modulus category.

When solving some applied problems, the classic methods of mathematics are not enough. Therefore, various non-traditional theories related to the solution of these problems were set up. The first non-traditional theory, the theory of fuzzy sets related to social problems and computer programing was created by Lotfi Zadeh in 1965. On one hand, this theory has laid foundation of very valuable mathematical logic, on the other hand opened wide horizons for development of computer technology. Based on Lotfi Zadeh's this work, later the theories of intuitionistic sets, rough sets, soft sets were set up and they were used in logical programming, medical diagnostics, decision-making, etc. The theory of fuzzy sets are used almost in all the fields of mathematcs as topology, algebra, geometry, functional analysis and so on.

The dissertation work was devoted to the fuzzy logic theory founded by Lotfi Zadeh and problems in its application in practice.

Beginning from 1969 Chang.C.L began to apply the theory of fuzzy sets in topology and this theory penetrated all the fields of mathematics. These theories were mainly applied in topology and algebra and formed new fuzzy topology and fuzzy algebra fields in mathematics. To present, there were some studies on fuzzy topology especially in the field of general topology, but such researches were not conducted in such an important field of topology as algebraic topology. In the same way, homological algebraic methods were not studied enough in fuzzy algebra. The main reason is inability to establish homological theory of fuzzy modulus space. The dissertation work studies unsolved problems and therefore the topic of the dissertation work is urgent and allows to future research works.

In 1969, for the first time fuzzy sets in topology were studied by Chang. Later, some scientists as Bayramov S.A., Li S.G., Zahedi M.M. were engaged in transfer of the results of general topology to fuzzy topological spaces. These results were reflected in Ying-Ming's book. In 1971 Rozenfied has applied fuzzy sets in topology and then was introduced to a fuzzy ring, modulus, Lie algebra and other structures and some researches were conducted. In his paper Zahedi, Ameri for the first time applied the methods of homological algebra to fuzzy sets. Generalization of fuzzy sets were included in intuitionistic fuzzy sets by Atanasov. Neutrosophic sets as a generalization of intuitionistic fuzzy sets were given in F.Smarandache's paper. Soft sets theory with its great application was build by Molotsov in 1999. Maji and Roy had great services in researching these sets.

Later fuzzy and soft structures were combined and fuzzy soft sets were structured. Application of soft sets in algebra began in 2007 by N.Cagman. He has given the notion of soft group, soft rings, soft modulus and studied some properties of these structures. Intuitionistic soft modulus were introduced in algebra by Gunduz Aras C., Bayramov S.A. Here the greatest problem in related to the setting of homotopy. In his paper Saleh conducted researches related to homotopy. Topological group of fuzzy topological spaces was structured. But the homothopy given here is not an equivalence relation. Singular homological theory in soft topological spaces was completely established.

Soft sets in topology were studied in 2011 and some researches related to soft topology in this fied were conducted. Note that the results in fuzzy sets were obtained mainly on general topology. But a strong tool as the methods of algebraic topology was not studied in these researches. In this field we can mention some papers.

On the other hand, when constructing some new category, there arises a problem of cloure of categories with respect to algebraic operations. Since direct and inverse limits contain all algebraic operations, the closure problem in these categories can be solved by showing the existence of direct and inverse limits. Some scientists have devoted their papers to the existence of such limits. For example, S.H.Lin's paper on fuzzy topolical space, M.Gadiri, B.Davvaz's paper in H- modulus category, V.Leoreanun's papers were devoted to the study of existence of inverse limit in SHR semi-group categories.

This dissertation work was devoted to the solution of some of these mentioned problems.

The goal and tasks of the study. The dissertation work was devoted to the solution of the problem of closure with respect to algebraic operations by means of direct and inverse categories in the structured new categories, to the study of the properties of functors and application of algebraic topology methods.

Research methods. In the problem under consideration methods of direct, inverse limits and closure method were used.

The main thesis to be defended.

- To prove theorems on universal coefficients by introducing derivative functor of tensor product in soft modulus category. To construct exact sequence of homological modules for the first time in the fuzzy modulus category and neutrospheric soft modulus category, to prove theorems on universal coefficients.
- Taking into account an important role of algebraic topology in mathematics, soft homological groups are constructed and it is proved that here the axioms of homological theory are satisfied.
- In each category there arises a problem of closure with respect to algebraic operations. Taking this into account, since direct and inverse limits contain all algebraic operations the existence of these limits in eac category plays an important role. To show the existence of direct and inverse limits in the categories of modulus, intuitionistic soft modulus, neutrospheric soft modulus. In addition, to construct exact sequence of the inverse limit related to the derivative functor.
- To construct neutrosophic Lie algebras and study their main properties.

Scientific novelty of the study. The following scientific novelties were obtained:

- To prove theorems on universal coefficients by introducing derivative functor of tensor product in soft modulus category. To construct exact sequence of homological modules for the first time in the fuzzy modulus category and neutrospheric soft modulus category, to prove theorems on universal coefficients.

- Taking into account an important role of algebraic topology in mathematics, soft homological groups are constructed and it is proved that here the axioms of homological theory are satisfied.
- In each category there arises a problem of closure with respect to algebraic operations. Taking this into account, since direct and inverse limits contain all algebraic operations the existence of these limits in eacg category plays an important role. To show the existence of direct and inverse limits in the categories of modulus, intuitionistic soft modulus, neutrospheric soft modulus.In addition, to construct exact sequence of the inverse limit related to the derivative functor.
- To construct neutrosophic Lie algebras and study their main properties.

Theoretical and practical importance of the study. Since the dissertation work was devoted to the fuzzy sets theory founded by Lotfi Zadeh and to the problems in its application in practice and homological methods are applied in fuzzy algebra, the topic of the work is urgent from theoretical and practical point of view and allows for future research.

Approbation and application. The main thesis of the dissertation work were reported in the conference devoted to the famous mathematician Majid Rasulov's 100-th anniversary, in the conference devoted to 91-th anniversary of the National leader Haydar Aliyev, "Actual problems of mathematics and mechanics" of yound doctoral students and young researchers, the conference "Functional analysis and its applications" devoted to honored scientist of Azerbaijan, prof. Amir Shamil oglu Habibzade, in VIII International Euroasian conference, in the IX International conference of the Georgian Mathematical Union, etc.

Author's personal contribution. All the obtained results and proposals belong to the author.

Author's publications. The full text of the work was published in author's 10 scientific works. The list of papers is at the end of the thesis.

The name of organization where the dissertation work was executed. The dissertation work was executed in the chair of "Algebra and Geometry" at Baku State University.

Total volume of the dissertation work indicating the volume of structural section of the dissertation separately in signs. The dissertation work consists of introduction, three chapters, results, a list of references of 76 titles. Total volume of the work – 234.760 signs (title page– 471 signs, table of contents – 1.706 signs, introduction -36.583 signs, chapter I – 7.000 signs, chapter II – 8 000 signs, chapter III –46.000 signs).

THE CONTENT OF THE DISSERTATION WORK

In the introduction the rationale of the work is justified, short abstract of results related to the dissertation topic is given and the main results are explained.

In chapter I universal coefficients theorems in soft modulus and fuzzy soft modulus category are proved. At the end of chapter I the Chekh homological theory is built in the category of soft topological spaces. Application of algebraic topology in fuzzy sets is also given in chapter I.

In *section 1.1* universal coefficients theorems in soft modulus category were proved. In *section 1.2* a universal coefficient theorem in the category of fuzzy soft modulus was given. In *section 1.3* the Chekh homological theory of soft topological spaces was built. *Section 1.4* deals with universal coefficient theorem in the category of neutrosophic modulus.

Chapter II considers closure problems in the category of fuzzy modulus. In *section 2.1* the derivative functor of the inverse limit in the category of soft modulus is given and some properties of the inverse limit are studied. In *section 2.2* the notion of inverse system in the category of intuitionistic fuzzy soft modulus is given and some theorems related to it is proved. In *section 2.3* a derivative functor of the $\varprojlim$ functor in the category of neutrosophic soft modulus is given, the existence and properties of the inverse limit in these categories are studied. In *section 2.4* a theorem on the existence of direct limit in some categories are proved.

In chapter III the Lie algebras and neutrosophi soft Lie algebras are studied. In *section 3.1* neutrosophic Lie algebras and their properties are studied. In *section 3.2* neutrosophic soft Lie algebras and their properties are studied.

In *section 3.3* same catagories prove theorius about the existence of a straight limit.

Definition 1. Let (F, A) on M and (G, B) on N be soft modules. For the functor $Tor((F, A),(G, B))$ we give

$Tor(F(a)),(G(b))$ and call the functor $Tor((F,A),(G,B))$ a derivative functor of the tensor product $(F \otimes G, A \times B)$ $\forall (a,b) \in A \times B$.

Let $(F,A) = \{((F_n,A),\partial_n 1_A) : (F_n,A) \to (F_{n-1},A)\}$ be a chain complex of soft modulus on the modules $\{M_n\}$ and (G,B) be a soft modulus on N. Then

$$\{(F_n \otimes G, A \times B),(\partial_n \otimes 1_G, 1_{A\times B})\} \tag{1}$$

is a soft chain complex on the modules $\{M_n \otimes N\}$. Here for

$$\forall (a,b) \in A \times B$$

$$\{F_n(a) \otimes G(b), \partial_n \otimes 1_{G(b)} : F_n(a) \otimes G(b) \to F_{n-1}(a) \otimes G(b)\} \tag{2}$$

is a chain complex of the modules.

Show the homological modulus of the complex (2) in the form of $H_n(F(a):G(b))$. So, for $\forall (a,b) \in A \times B$ by means of the modulus $H_n(F(a);G(b))$ we can give soft homological modulus of the soft modulus of chain complex (1). Denote this modulus by $H_n(\tilde{F},A);(G,B)$ and call a homological modulus with the coefficient (G,B) of the complex (F,A).

We define homomorphism of soft modules

$$(\mu, 1_{A\times B}) : H_n(\tilde{F},A) \otimes (G,B) \to H_n(\tilde{F},A);(G,B)$$

for $\forall (a,b) \in A \times B$ as

$$\mu_{(a,b)} : H_n(F(a)) \otimes (G(b)) \to H_n(F(a)),(G(b))$$

$$\mu_{(a,b)}([x],g) = [x \otimes g] \;\; \forall [x] \in H_n(F(a)), g \in G(b).$$

Theorem 1. Let the chain complex $(\tilde{F},A)$ be a complex of independent soft modules on the modull $\{M_n\}$ and (G,B) be an arbitrary soft modulus on N. Then the following sequence is exact, functorial and seperable

$$0 \to (H_n(\tilde{F},A)) \otimes (G,B) \xrightarrow{\mu} H_n((\tilde{F},A) \otimes (G,B)) \to$$
$$\to T_{or}(H_{n-1}(\tilde{F},A)(G,B)) \to 0$$

Theorem 2. if the complex of chains $(F,A) * (G,B)$ is acyclic, then the short sequence

$$0 \to H_n(F,A) \otimes (G,B) \xrightarrow{\mu} H_n(.(F_1,A):(G,B)) \to$$
$$\to H_n(F,A) * (G,B) \to 0$$

is exact, functorial and separable.

Assume that $\forall n \in Z(F_n, A)$ is a fuzzy soft modulus on the modulus M_n and $(\partial_n, 1_A):(F_n,A) \to (F_{n-1},A)$ is an homomorphism of the fuzzy soft modules.

Definition 2. If for each $a \in A$

$$\{(M_n, F_n(a)), \partial_n : (M_n, F_n(a)) \to (M_{n-1}, F_{n-1}(a))\}$$

is a complex of chains of fuzzy modules, then the following sequence is said to be a complex of chains of fuzzy soft modules:

$$\{(F_n, A), (\partial_n, (I_A)) : (F_n, A) \to (F_{n-1} A)\}$$

Definition 3. Assume that

$$(\{\varphi_n\}, g) : (\{\psi_n\}, g) : \{(F_n, A), \partial_n\} \to \{(G_n, B), \partial'_n\}$$

is a morphism of the fuzzy soft modules and assume that is $D = \{(D_n, g) : (F_n, A) \to (G_{n+1}, B)\}$ is a family of fuzzy soft modules. If the expression $\varphi_n - \psi_n = D_{n-1} \circ \partial_n + \partial'_{n+1} \circ D_n$ is satisfied, then the homotopy of homomorphisms of modulus

$$D = \{(D_n, g) : M_n \to N_{n+1}\}_{n \in Z}$$

is said to be a chain family. The pairs $(\{\varphi_n\}, g), (\{\psi_n\}, g)$ is called chain homotopy morphisms and is denoted as $(\{\varphi_n\}, g) \sim (\{\psi_n\}, g)$.

Theorem 3. In the category of fuzzy soft modulus, the chain homotopy relation is an equivalence relation and is invariant with respect to superposition.

Theorem 4. The homological functor of chain complexes of fuzzy soft modulus is invariant with respect to chain homotopy. Therefore, if $\{\varphi_n\} \sim \{\psi_n\} : \{(F_n, A), \partial_n\} \to \{(G_n, B), \partial'_n\}$, then $\varphi_{n*} = \psi_{n*} = H_n(\mathscr{F}, A) \to H_n(G, A)$

Theorem 5. If this sequence

$$0 \to (F'_n, A) \to (F_n, A) \to (F''_n, A) \to 0 \qquad (3)$$

the short exact sequence of fuzzy soft chain complexes is separable, then the following sequence of fuzzy soft homological modulus is exact:

$$\cdots \leftarrow H_{n-1}(F_n', A) \xleftarrow{\partial_*} H_n(F_n'', A) \leftarrow H_n(F_n, A) \leftarrow H_n(F_n', A) \cdots \quad (4)$$

Theorem 6. Each separable short exact sequence of fuzzy soft chain complex

$$0 \to (F', A) \to (F, A) \to (F'', A) \to 0$$

and for each (G, B) fuzzy modulus, the sequence of fuzzy soft homological modules

$$\cdots \leftarrow H_{n-1}((\mathscr{F}', A);(G, B)) \leftarrow H_n((F'', A);(G, B)) \leftarrow$$
$$\leftarrow H_n((\mathscr{F}, A);(G, B)) \leftarrow H_n((F_n', C');(G, B)) \leftarrow \cdots$$

is exact and functorial.

Theorem 7. If (F, A) is an independent fuzzy soft chain complex and (G, B) is a fuzzy soft modulus, then

$$0 \to H_n((F, A) \otimes (G, b)) \xrightarrow{\overline{\varphi}_n} H_n((F, A);(G, B)) \to$$
$$\to FS - Tor(H_{n-1}((F, A),(G, b))) \to 0$$

is short, exact functorial and separable.

In this section we build up a homological theory in *Stop* category. Let us denote by *Stop* a category of soft topological spaces. Let for each soft topological space (X, τ, E) $Cov(X)$ be a set of all open covers of this space. $Cov(X)$ is a set directed to refinement the covers. Let the families $\alpha = \{(F_i, E)\}_{i \in I}$ and $\beta = \{(G_i, E)\}_{i \in J}$ be open covers for soft the soft topological space (X, τ, E). If for the mapping $p: J \to I$ the condition $(G_j, E) \subset (F_{p(j)}, E)$ is satisfied, the cover α is called β narrowing of the covers. Show it as $\alpha < \beta$. The set $Cov(X)$ is a set directed according to this relation. Let the family $\alpha = \{(F_i, E)\}_{i \in I}$ be an arbitrary open cover of the soft topological space (X, τ, E). By

$$nerv\alpha = \left\{ (i_1, i_2, \ldots, i_n) : \bigcap_{k=1}^{n} \left(F_{I_K} E \right) \neq \Phi \right\}$$

we denote a simplical complex whose vertices are points of the set I. The mapping $p : J \to I$ determines the simplicial mapping $p : nerv\beta \to nerv\alpha$ and such arbitrary two mappings are simplicially close. So, we determine the simplicial mapping $p_\alpha^\beta : nerv\beta \to nerv\alpha$ And we get the inverse system of the simplicial complexes

$$nerv(X) = \left(\{nerv\alpha\}_{\alpha \in \mathrm{cov}(x)}, \{p_\alpha^\beta : nerv\beta \to nerv\alpha\}_{\alpha < \beta} \right)$$

Applying to this system the homological functor H_q we get the inverse (direct) system of the groups

$$H_q(nervX) = \left(\{H_q(nervX)\}_{\alpha \in \mathrm{cov}(x)}, \{H_q\left(p_\alpha^\beta\right) : H_q(nerv\beta) \to \right.$$
$$\left. \to H_q(nerv\alpha)\} \alpha < \beta \right)$$

$$\left[H^q(nervX) = \left(\{H^q(nerv\alpha)_{\alpha \in \mathrm{cov}(x)}, \{H^q\left(p_\alpha^\beta\right) : H^q(nerv\alpha)\} \to \right. \right.$$
$$\left. \left. \to H^q(nevv\beta) \alpha < \beta \} \right) \right].$$

Definition 4. The group

$$H_q(X;G) = \lim_{\overleftarrow{\alpha}} H_q(nerv\,\alpha; G) \left[H^q(X;G) = \lim_{\overleftarrow{\alpha}} H^q(nerv\,\alpha; G) \right]$$

is a $q-$ dimensional homological (cohomological) group of the soft topological space (X, τ, E).

If (X, τ, E) and (Y, τ^1, E^1) are soft topological spaces and $(f, \varphi) : (X, \tau, E) \to (Y, \tau^1, E^1)$ is a soft continuous function of the soft topological spaces for arbitrary soft open cover $\alpha = \{G_j, E^1\}_{j \in J}$ of the space (Y, τ^1, E^1) the family $(f, \varphi)(\alpha) = \{(f, \varphi)^{-1} F_j, E^1\}_{j \in J'}$ is a soft open cover of the space (X, τ, E) and $J' \subset J$. İt is clear that if $\beta > \alpha$ then $(f, \varphi)^{-1}(\beta) > (f, \varphi)^{-1}(\alpha)$ and if

$$\left((f, \varphi)^{-1} \right) \{G_j, E^1\} \cap \cdots \cap \left((f, \varphi)^{-1} \right) \{G_{jk}, E^1\} \neq \Phi$$

is satisfied, then

$\{G_j, E^1\} \cap \cdots \cap \{G_{jk}, E^1\} \neq \Phi$. Hence, the simplicial complex $nerv\left((f,\varphi)^{-1}\right)(\alpha)$ is a subcomplex of the simplicial complex $nerv\,\alpha$. By $i_{\alpha,(f,\varphi)} : nerv\left((f,\varphi)^{-1}\right)(\alpha) \to nerv\,\alpha$ show the inclusion maping, then the family

$$f = \left(\left\{(f,\varphi)^{-1} : Cov(Y) \to Cov(X)\right\},\right.$$
$$\left.\left\{i_{\alpha,(f,\varphi)} : nervf\left((f,\varphi)^{-1}\right)(\alpha) \to nerv\,\alpha\right\}_{\alpha \in Coc(Y)}\right), \tag{5}$$

is a morphism acting on the inverse system $nerv(X)$ from the inverse system $nerv(X)$. The f morphism determines the homomorphism of the homological (cohomological) groups

$$f_* = \lim_{\leftarrow} H_q(f) : H_q(X,G) \to H_q(Y,G)$$
$$\left[f_* = \lim_{\leftarrow} H_q(f) : H^q(Y,G) \to H^q(X,G)\right]$$

Theorem 8. The $(X,\tau,E) \mapsto H_q(X,G)\left[(X,\tau,E) \mapsto H^q(X,\tau,E) \mapsto\right]$ is a covariant (countervariant) functor going from the $STop$ category to the group category.

Theorem 9. For each soft point $x_b \in (X,\tau,E)$

$$H_q(x_b, G) = \begin{cases} 0, & q > 0 \\ G, & q = 0. \end{cases}$$

For the pairs of soft topological spaces (X, A, τ, E) we take the mapping $i : A \to X$ and $j : X \to (X, A)$.

For each cover $\alpha \in Cov(X, A)$ from the mapping i, j we obtain the simplicial mappings $i_\alpha : nerv(\alpha \cap A) \to nerv\,\alpha$,

$$j_\alpha : nerv\alpha \to \left(nerv\,\alpha, nerv\left(\alpha \cap \tilde{A}\right)\right).$$

So, for each $\alpha \in Cov(X, A)$ we get exact sequence of homological groups

$$H(nerv\alpha) = \cdots \leftarrow H_q(nerv\alpha) \leftarrow H_q\left(nerv\left(\alpha \cap \tilde{A}\right)\right) \leftarrow$$
$$H_{q+1}\left(nerv\alpha, nerv\left(\alpha \cap \tilde{A}\right)\right) \leftarrow H_{q+1}(nerv\alpha) \leftarrow \cdots$$

These sequences create an inverse system with respect to α. The limit of this inverse system is said to be a homological sequence of the pair (X, A, τ, E):

$$\cdots H_q(X,G) \leftarrow H_q(A,G) \leftarrow H_{q+1}(X,A,G) \leftarrow H_{q+1}(X)\cdots$$

Since the inverse limit of exact sequence do not have an exact sequence, the homological sequence is not exact, but the cohomological sequence

$$\cdots \rightarrow H^q(X,G) \rightarrow H^q(A,G) \rightarrow H^{q+1}(X,A,G) \rightarrow H^{q+1}(X,G)\cdots$$

is exact.

Theorem 10 (Cutting axiom). Assume that (X, A, τ, E) is a soft topological space, $(U,E) \in \tau$ and its soft closure $\overline{(U,E)}$ are in soft inside of A i.e.let $\overline{(U,E)} \subset \widetilde{A}^0$. Then for the inclusion mapping $J : (X-U, A-U) \rightarrow (X, A)$ the homomorphism

$$J_{*q} : H_q(X-U, A-U; G) \rightarrow H_q(X, A; G)$$

$$\left[J^{*q} : H_q(X-U, A-U; G) \rightarrow H^q(X, A; G)\right]$$

is an isomorphism.

Definition 5. Assume that, (X, τ, E) and (Y, τ', E') are two soft spaces and $(f, \varphi), (g, \varphi) : (X, \tau, E) \rightarrow (Y, \tau', E')$ is a continuous mapping of soft topological spaces. If there exists a soft continuous mapping $(F, \varphi) : (X \times I, \tau \times \tau', E \times (*)) \rightarrow (Y, \varepsilon', E')$ satisfying the condition,

$$(F,\varphi)(x_b, O_*) = (f,\varphi)(x_b) = f(x)_{\varphi(b)}$$

$$(F,\varphi)(x_b, 1_*) = (g,\varphi)(x_b) = g(x)\varphi(e)$$

then (F, φ) is called a homotopy, the mapping $(f, \varphi), (g, \varphi)$ are said to be homotopic mapping .

İt is clear that the homotopy relation is an equivalence relation and is invariant with respect to superposition.

Theorem 11 (*Homotopy axiom***).**

If $(f, \varphi), (g, \varphi) : (X, \tau, E) = (Y, \tau', E')$ are soft homotopic mapping, then

$$(f,\varphi)_* = (g,\varphi)_* : H_q(X,G) \rightarrow H_q(Y,G)$$

is valid.

In chapter II we study the closure problem in various categories.

Definition 6. Let $SMod$ be a category of soft modulus. Then the functor $D: I^{0p} \to SMod$ $(D: I \to SMod)$ is said to be an inverse (direct) system.

According to definition, we can write each inverse system in the form of

$$\{(F_i, A_i)\}_{i\in I},\ \{(p_i^{i'}, q_i^{i'}): (F_{i'}, A_{i'}) \to (F_i, A_i)\}_{i\in i'} \tag{6}$$

and the following conditions are satisfied:

1) $i = i'$ for $(p_i^{i'}, q_i^{i'}) = 1_{(F_i, A_i)}$;

2) $i < i' < i''$ for $(p_i^{i''}, q_i^{i''}) = (p_{i'}^{i''}, q_{i'}^{i''}) \circ (p_i^{i'}, q_i^{i'})$.

Theorem 12. Every inverse system in the form of (6) has a limit and it is unique.

Theorem 13. In the confration $\{(F_i, A_i)\}_{i\in I}$, $\{(p_i^{i'}, q_i^{i'})_{i<i'} \to \varprojlim (F_i, A_i)\}$ is a functor acting from the category $Jnv(SMod)$ to the category $SMod$.

Now let us define the homomorphism $d: \prod_i M_i \to \prod_i M_i$ in the form

$$d(\{x_i\}) = \{x_i - p_i^{i'}(x_{i'})\}.$$

It is clear that for, $\forall a \in A$

$$d(a) = d|_{\prod_i F(a)} : \prod_i F(a) \to \prod_i F(a)$$

is an homomorphism of appropriate moduless. Then we can give the modulus $\ker d(a)$ and $co\ker d(a)$. It is clear that, $\ker d(a) = \varprojlim F_i(a)$. For each $a \in A$ given by $co\ker d(a)$ can be accepted as a soft modulus on the modulus $\prod_i M_i$. Show this soft modulus as $\varprojlim^{(1)}(F_i, A)$ and call this modulus the first derivative functor of the inverse limit factor.

So, we get the equality $\varprojlim_i (F_i, A) = \ker d$, $\varprojlim_i^{(1)} (F_i, A) = co\ker d$.

Proposition 1. The soft modulus $\varprojlim_i^{(1)}$ is a funktor acting from the category of inverse systems of soft modulus to the category of soft modulus.

Proposition 2. $\varprojlim_i (F_\alpha, A) = H^0(C)$, $\varprojlim_i^{(1)} (F_i, A) = H^1(C)$.

Let us study some properties of the funktor $\varprojlim_i^{(1)}$.

From the directed set I we get the set of natural numbers N. Then the inverse system will be in the form

$$(F_1, A) \underset{p_1^2}{\leftarrow} (F_2, A) \underset{p_2^3}{\leftarrow} \ldots.$$

Theorem 14. For each infinite subsystem in the inverse system of soft modulus $(F_1, A) \overset{p_1^2}{\leftarrow} (F_2, A) \overset{p_2^3}{\leftarrow} \ldots$ the $\varprojlim^{(1)}$ factor is invariable.

Theorem 15. If

$$(F_1, A) \overset{p_1^2}{\leftarrow} (F_2, A) \overset{p_2^3}{\leftarrow} \ldots$$

In the inverse *SMod* system p_i^{i+1} homomorphisms are epimorphisms, then $\varprojlim^{(1)} (F_n, A) = 0$.

Theorem 16. Let

$$\begin{array}{ccccccccc}
 & & \vdots & & \vdots & & \vdots & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & (F_2', A) & \to & (F_2, A) & \to & (F_2'', A) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & (F_1', A) & \to & (F_1, A) & \to & (F_1'', A) & \to & 0
\end{array}$$

be a short exact sequence of the inverse systems of soft modulus. Then the sequence

$$0 \to \varprojlim(F_n', A) \to \varprojlim(F_n, A) \to \varprojlim(F_n'', A) \to$$
$$\to \varprojlim^{(1)}(F_n', A) \to \varprojlim^{(1)}(F_n, A) \to \varprojlim^{(1)}(F_n'', A) \to 0$$
of the soft modulus is exact

Definition 7. Let IGSM be be an category of intuitionistic fuzzy soft modulus, then $D : \Lambda^{op} \to IQSM$ is called an inverse system of fuzzy soft modulus to the functor Λ.

Theorem 17. Every inverse system of intuitionistic fuzzy soft modulus has a limit and this limit is unique.

For the inverse system of modulus $\left(\{M_\alpha\}_{\alpha\in\Lambda}, \{p_\alpha^{\alpha'}\}_{\alpha\prec\alpha'}\right)$,

$\lim\limits_{\leftarrow}^{(1)} M_\alpha = \prod\limits_\alpha M_\alpha \Big/ \operatorname{Im} d$ is a derivative function.

If $\pi = \prod\limits_\alpha M_\alpha \to \lim\limits_{\leftarrow}^{(1)} M_\alpha$ is a canonical homomorphism, then we can determine the intuitionistic fuzzy modulus $\left(\lim\limits_{\leftarrow}^{(1)} M_\alpha, (F_A)_\alpha^\pi, (F_A)_\pi^a\right)$. Then $\left(F_A^\pi, F_\pi^A\right) : A \to \prod\limits_\alpha M_\alpha$ is an intuitionistic fuzzy soft modulus.

Definition 8. $\left((F_A)^\pi, \left(F^A\right)_\pi\right)$ is called “the first derivative factor” of intuitionistic fuzzy soft modulus of the inverse system.

Theorem 18. Assume that the sequence
$$(F_1, A) \xleftarrow{p_1^2} (F_2, A) \xleftarrow{p_2^2} \ldots$$
is the inverse sequence of intuitionistic fuzzy soft modulus. For each infinite subsequence of this sequence $\varprojlim^{(1)}$ does not change.

Theorem 19. It for all $\{x_n''\} \in \ker \bar{d}$ or $\lim\limits_{n\to\infty} F_{na}''(x_n'') = 0$ and the $\lim\limits_{n\to\infty} F_n''^{a}(x_1'') = 1$ the following diagram is a short exact sequence of the inverse system of fuzzy soft modulus

$$\begin{array}{ccccccccc}
 & & \vdots & & \vdots & & \vdots & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & (F_2', A) & \to & (F_2, A) & \to & (F_2'', A) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & (F_1', A) & \to & (F_1, A) & \to & (F_1'', A) & \to & 0
\end{array}$$

then the sequence

$$0 \to \varprojlim(F'_n, a) \to \varprojlim(F_n, a) \to \varprojlim(F''_n, a) \to$$
$$\varprojlim{}^{(1)}(F'_n, a) \to \varprojlim{}^{(1)}(F_n, a) \to \varprojlim{}^{(1)}(F''_n, a) \to 0$$

is exact.

Definition 9. A neytrosophic set on the set X is determined as follows A:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where

$$T, I, F : X \to \left]^{-}0, 1^{+}\right[\text{ və } -0 \le T_A(x) + I_A(x) + F_A(x) \le +3$$

In this section is given the notion of derivative functor of the inverse limit in the category of neutrosopic soft modulus.

Definition 10. Any functor of each $D : \Lambda^{op} \to NSM$ is said to be an inverse system of neutrosophic soft modulus. Here Λ is a directed set.

Theorem 20. Every inverse system of neutrosophic soft modulus has a limit and this limit is unique.

Definition 11. If for the neutrosophic set $A = (T, I, F)$ given on the Lie algebra the following conditions are satisfied, then $A = (T, I, F)$ is said to be a neutrosophic Lie subalgebra. For all $x, y \in L$ and $\alpha \in F$ taken from the set L

(1) $T_A(x+y) \ge \min(T_A(x), T_A(y))$

$I_A(x+y) \ge \min(I_A(x), I_A(y))$

$F_A(x+y) \le \max(F_A(x), F(y))$

(2) $T_A(\alpha x) \ge T_A(x)$, $I_A(\alpha x) \ge I_A(x)$, $F_A(\alpha x) \le F(x)$

(3) $T_A([x, y]) \ge \min\{T_A(x), T_A(y)\}$

$I_A([x+y]) \ge \min\{I_A(x), I(y)\}$

$F_A([x+y]) \le \max\{F_A(x), F(y)\}$

Theorem 21. Assume that, $A = (T, I, F)$ is a neutrosophic Lie subalgebra on the Lie algebra L. Then $A = (T, I, F)$ is a neutrosophic Lie subalgebra on L if and only if for non-empty high level

$$U_T(s)=\{x\in L|T(x)\geq s\},\ U_I(s)=\{x\in L|I(x)\geq s\}$$

and non-empty low level $V_F(s)=\{x\in L|F(x)\leq t\}$ is Lie subalgebra of L for all $s,t\in[0,1]$.

Theorem 22. If $A=(T_A,I_A,F_A)$ and $B=(T_B,I_B,F_B)$ are two Lie subalgebras on L then the intersection $A\cap B=C=\langle T_C,I_C,F_C\rangle$ is a Lie subalgebra on L.

Definition 12. Assume that $A=(T^1,I^1,F^1)$ and $B=(T^2,I^2,F^2)$ are two neutrosophic sets given on the set L. The generalized direct product $A\times B$ is determined as follows:

$$A\times B=(T^1,I^1,F^1)\times(T^2,I^2,F^2)=(T^1\times T^2,I^1\times I^2,F^1\times F^2),$$

here

$$(T^1\times T^2)(x,y)=\min(T^1(x),T^2(y)),$$

$$(I^1\times I^2)(x,y)=\min(I^1(x),I^2(y))$$

and

$$(F^1\times F^2)(x,y)=\max(F^1(x),F^2(y)).$$

We can note that the generalized direct product $A\times B$ is always neutrosophic in $L\times L$, so that

$$\min(T^1(x),T^2(y))+\min(I^1(x),I^2(y))+\max(F^1(x),F^2(y))\leq 3.$$

Theorem 23. Assume that $A=(T^1,I^1,F^1)$ and $B=(T^2,I^2,F^2)$ are two neutrosophic Lie subalgebras of the Lie algebra L. Then $A\times B$ is a neutrosopic Lie subalgebra of $L\times L$.

Definition 13. Assume that L_1 and L_2 are two Lie subalgebras on the domain F. The the linear map $f:L_1\to L_2$ is called Lie homomorphism, if for all $x,y\in L_1$ the $f([x,y])=[f(x),f(y)]$ is satisfied.

Theorem 24. Assume that $f:L_1\to L_2$ is epimorphism of the Lie algebras and $A=(T,I,F)$ is a neutrosophic subalgebra of L_1, then a homomorphic image of A is neytrosophic subalgebra of L_2.

Definition 14. Assume that E is the set of all parameters, L is a Lıe algebra, $P(L)$ are all neutrosophic sets fixed on L. Then the pair $(\widetilde{F}, E)$ is called a neutrosophic soft Lie algebra on L_1 here $\widetilde{F}: E \to P(L)$ is an acting map, so for $\forall e \in E$ the conditions of definition 11 are satisfied.

Theorem 25. Assume that $(\widetilde{F}^1, E_1)$ and $(\widetilde{F}^2, E_2)$ are two neutrosophic soft Lie algebras on L, then $(\widetilde{F}^1, E_1) \cap (\widetilde{F}^2, E_2) = (\widetilde{F}^3, E_1 \cap E_2)$ is a neutrosiphic soft Lie subalgebra on L.

Theorem 26. Assume that $(\widetilde{F}^1, E_1)$ and $(\widetilde{F}^2, E_2)$ are two neutrosophic soft Lie subalgebras on L. If $E_1 \cap E_2 = \varnothing$, then $(\widetilde{F}^1, E_1) \cup (\widetilde{F}^2, E_2) = (\widetilde{F}^3, E_1 \cup E_2)$ is a neutrosophic soft Lie subalgebra on L.

Theorem 27. Assume that $(\widetilde{F}^1, E_1)$ and $(\widetilde{F}^2, E_2)$ are two neutrosophic soft Lie subalgebras on L_1 and L_2 respectively, then $(\widetilde{F}^1, E_1) \wedge (\widetilde{F}^2, E_2) = (\widetilde{F}^3, E_1 \cup E_2)$ is a are neutrosophic soft Lie algebra on L.

Definition 15. Let $(\widetilde{F}^1, E_1)$ and $(\widetilde{F}^2, E_2)$ be two neutrosophic soft sets in the set L, then the common direct product $(\widetilde{F}^1, E_1) \times (\widetilde{F}^2, E_2) = (\widetilde{F}^1 \times \widetilde{F}^2, E_1 \times E_2)$ is determined as follows:

$$\widetilde{F}^1 \times \widetilde{F}^2 : E_1 \times E_2 \to NS(L)$$

$$\widetilde{F}^1 \times \widetilde{F}^2 (e_1 e_2) = \left(T_{p^1}(e_1) \times T_{p^2}(e_2)\right), \left(I_{p^1}(e_1) \times I_{p^2}(e_2)\right),$$

$$F_{p^1}(e_1) \times F_{p^2}(e_2)$$

here for every $(e_1 e_2) \in E_1 \times E_2$ $F_1(e_1) \times F_2(e_2)$ satisfies the condition of definition 12.

Theorem 28. Let $(\widetilde{F}^1, E_1)$ and $(\widetilde{F}^2, E_2)$ be two neutrosophic soft Lie subalgebras on L, $(\widetilde{F}^1, E_1) \times (\widetilde{F}^2, E_2)$ is a neutrosophic soft Lie subalgebra on $L \times L$.

Theorem 29. Assume that $f : L_1 \to L_2$ is an homomorphism of Lie algebras and $(\tilde{F}, E)$ is a neutrosophic soft Lie subalgebra of L_1, then the homomorphic image of $(\tilde{F}, E)$ is a neutrosophic soft Lie subalgebra of L_2 .

Theorem 30. $Autg$ automorphism group of the Lie algebra g is a structural group of the stratification $L = (L, p, B)$[1].

Here we will give the relation between the stratification $L = (L, p, B)$ and tangent TM by means of the Lie algebra g . Just this relation is obtained from the inclusion of the layer g to the followoing exact sequence:

$$0 \to Zg \to g \to g_0 \to 0.$$

Theorem 31. Automorphism group $Autg$ acts invariantly on g . Action of the center $Z\ g$ on g_0 is invariant.

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[1]Gasymov, VA, Abdullaev, S.A. On some properties of groupoids and algebroids Lee // -Baku: Baku University news, -2018. №3, -p. 13-19.
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RESULTS

The dissertation work was devoted to the Lie algebroid and groupoids and properties of functors in the fuzzy moduluscategory.

The following scientific novelties were obtained:

- Theorems on universal coefficients by introducing derivative functor of tensor product in soft modulus category were proved.
- Exact sequence of homological modules was constructed for the first time in the fuzzy modulus category and neutrospheric soft modulus category, and using this theorems on universal coefficients were proved.
- Taking into account an important role of algebraic topology in mathematics, homolojucal group of soft topological spaces is constructed and the axioms of the homologous theory are proved.
- In each category there arises a problem of closure with respect to algebraic operations. Taking this into account, since direct and inverse limits contain all algebraic operations the existence of these limits in eacg category plays an important role. To show the existence of direct and inverse limits in the categories of modulus, intuitionistic soft modulus, neutrospheric soft modulus. In addition, to construct exact sequence of the inverse limit related to the derivative functor.
- Neutrosophic Lie algebras were constructed and their main properties were studied.

The main results of the dissertation work were published in the following works:

1. Abdullayev, S.E., Bayramov, S.A. Derivative of the inverse limit in the category of soft modules // -Baku: Baku University news, -2018. №1, -p. 24-32.
2. Abdullayev, S.E., Bayramov, S.A. Theorems on universal coefficients in the category of soft modules // -Lankaran: News of Lankaran University, -2018. №1, -p. 12-18.
3. Gasymov, VA, Abdullaev, S.A. On some properties of groupoids and algebroids Lee // -Baku: Baku University news, -2018. №3, -p. 13-19.

4. Abdullayev, S.E., Bayramov, S.A. Inverse system in the category of intuitionistic fuzzy soft modules // Journal of Advances in Mathematics, -2018. v.14. №1, -p. 7893-7904.
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10. Cigdem, G.A., Abdullayev, S.A. The Cech homology theory in the category of soft topological spaces // -Baku: Transactions of NAS of Azerb., -2020. v. 40, № 1, -p. 1-11.

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